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ABSTRACT

This article examines the effects of a secondary school mathematics curriculum. This curriculum has been developed by the Dutch National Institute for Curriculum Development (SLO). There are differences in how this curriculum is implemented in the classroom. There are striking differences in the learning results at pupil, class and school levels. Systematic instruction to the class as a whole is positively related to cooperation and task orientation in small groups. Task orientation is a relatively strong predictor of learning results as measured by the post-test. The conclusion is drawn that although in general the pupils make significant progress from pre-test to post-test, not all pupils attain a certain minimum level. Suggestions for improvements are provided. (Author/MNS)

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MATHEMATICS FOR ALL: BETWEEN DREAM AND REALITY

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ABSTRACT

This article examines the effects of a mathematics curriculum. This curriculum has been developed by the Dutch National Institute for Curriculum Development (SLO). There are differences in how this curriculum is implemented in the classroom. There are striking differences in the learning results at pupil, class and school level. Systematic instruction to the class as a whole is positively related to cooperation and task orientation in small groups. Task orientation is a relatively strong predictor of learning results as measured by the post-test. The conclusion is drawn that although in general the pupils make significant progress from pre-test to post-test, not all pupils attain a certain minimum level. Suggestions for improvements are provided.

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1. INTRODUCTION

In all countries of the industrialized world a debate is going on about "individual differences and the common curriculum" (Fenstermacher and Goodlad, 1983). In the field of mathematics education, Damerow and Westbury (1985) participate in this debate. According to their view there are two alternatives for the development of mathematics education. "Do we keep, for example, the highly selective frame works and methods of traditional mathematics education but give up the privileged position of the subject as a part of the core of general education? Or do we seek to keep mathematics at the core of the curriculum but find a way of teaching the subject to all students ?"

Although Damerow and Westbury do not expect great succes in the near future, they suggest to make "mathematics for all" the overall orientation of the subject.

This does not mean a reduction of the traditional curriculum to a "minimal competency" program by throwing overboard every mathematical idea and every difficulty. It is important to avoid the danger of one saying "mathematics for all is no mathematics at all" (de Lange, 1985). Mathematics for all implies a high achievement level. It implies also relevance to real-life situations. See also the article of Treffers and Goffree (1985) about mathematics education for 6 - 12 year olds.

Keitel (1985) tries to reconcile characteristics of mathematics for the majority with characteristics of a more advanced mathematics for a minority.
The fundament of this mathematics is what she calls the "Vorfeld der Mathematik" (mathematics in real life situations). In this "Vorfeld" the student applies rules unconsciously until he becomes aware of them. If he reflects upon his own problem-solving process he can reach a higher level of understanding. The results of this reflection may be the formulation of ideas and rules in general terms. This is what Freudenthal (1980) means by 'levels in the learning process'.

In this paper I will present the results of an evaluation of a math curriculum for secondary education. This curriculum may be seen as an experiment to teach mathematics to he majority, to reach a high level for all and to present mathematics in real life situations. Cooperative learning is an important aspect of this curriculum.

* This paper is based on research in our researchproject "Mixed Ability Teaching in Mathematics 12 - 16". (ID 12-16-SVO 0647) This project at the University of Utrecht is sponsored by the Foundation for Educational Research (SVO) in The Hague. I wish to acknowledge my great debt to Nathan Deen, Rijkje Dekker, Paul Herfs, Wytse de Jong, Dirk van der Ploeg and Karel Stokking for their support. I am also thankful to Jacobijn Sandberg and Jerry Andriessen for their help with translating this paper.

2. RESEARCH QUESTIONS

In the research project "Mixed ability teaching in Mathematics education for 12 - 16 year old pupils", we evaluate a new Mathematics curriculum. This curriculum has been developed by the (National) Foundation for Curriculum Development in the Netherlands.

The new curriculum is based on ideas of Freudenthal (1980) and Van Hiele (1982), but also on other recent (cognitive) theories about teaching and problem-oriented, cooperative learning.

The central thought behind the curriculum is "Mathematics for all". The written curriculum has the following main characteristics (Terwel, 1984):

1. working in heterogeneous classes by instruction and socratic discussion
2. cooperation in mixed ability small groups within the class
3. presenting Mathematics in real life situations
4. using the principle of levels in the learning process in relation to cooperation in mixed ability small groups.

The mathematical content of the curriculum consists of an introduction of relations and functions for thirteen year old pupils.

The research objective is to answer the following questions:

1. Will the common goals of this curriculum be reached?
2. If the goals have been are reached, why and how, if not, why not?

The specific research questions are: What is the relationship between (a) the student characteristics (b) the extent to which the characteristics of the curriculum have been implemented and (c) the extent to which learning has resulted?

3. THEORETICAL FRAMEWORK, METHODS AND SOURCE

The written curriculum is designed for different situations and realisations. On the basis of theoretical viewpoints and empirical findings in other research, the researchers formulated a central hypothesis:

In classes in which the teacher gives much systematic instruction and guidance and in which the students cooperate in a task-oriented way, learning results will be better than in classes with less teacher guidance and systematic instruction and with less task-oriented cooperation between the students.

The background of this hypothesis is formed by theories of guided discovery learning, time on task, cooperative learning and (cognitive) theories that underline the significance of prior knowledge.

From research we know that it is not useful to have small groups of students work without systematic instruction, feedback and guidance by the teacher. Central concepts, ideas and rules will not be learned and students will not use their time in an efficient manner. Especially, low performing students will be the victims of a 'laissez faire' teaching style (Brophy (1979 a and b) and Doyle, 1983). See also the discussion about discovery learning between Bruner and Ausubel (in Neber (1975). The following research model was used:

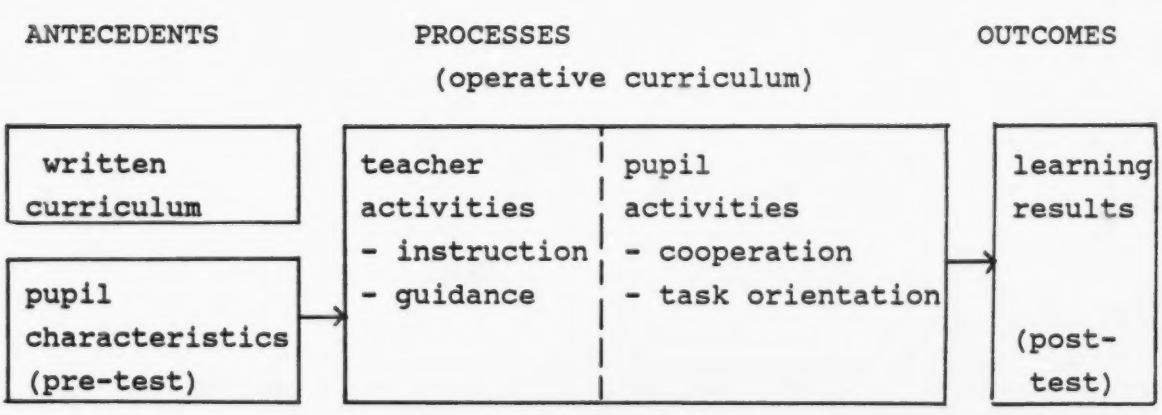

figure 1. *The research model in terms of antecedents, processes and outcomes.*

The Antecedents are measured by a curriculum specific pre-test on Mathematics.

The Processes are measured by a pupil questionnaire, the socalled PERCIA-scale (Perception of the Curriculum In Action). This scale is a curriculum specific modification of existing measures for perception of the learning environment. (Fraser, 1981)

The Outcomes are measured by a parallel version of the pre-test for the results in the cognitive domain.
About 375 pupils from a total of eighteen classes from two secondary schools, and six teachers are involved. The curriculum has been implemented from october until april.

School 1 is a 'comprehensive school' in a large urban city in the West of the Netherlands. This school has, as we shall see, relatively many low performing students. This is not surprising. This school has many students from families with low SES and from ethnic minorities.
School 2 is a secondary school in a middle large town in the East of the Netherlands.

4. PROCESSES

In order to obtain measures of what is going on in the 18 classes, we constructed the "PERCIA"-questionnaire, (PERception of the Curriculum In Action).
This questionnaire is similar to other scales for measuring the learning environment (Fraser 1981, Fischer and Fraser 1983). The idea behind the PERCIA is to describe the curriculum as seen through the eyes of the students. We asked the students to rate the frequency of activities in the classroom, for example:

	(ALMOST) NEVER	SELDOM	SOMETIMES	OFTEN	(ALMOST) ALWAYS
The students have quarrels in the groups					
This happens	1	2	3	4	5

To measure the activities of teachers and students four clusters of PERCIA-items were used;

a. Whole class instruction and discussion
The extent to which the teacher organizes introductions, reflections and discussions in the class as a whole.
Item example:
After the tasks are finished our teacher discusses them in class.

b. Guidance of small groups and individual students
The extent to which the teacher gives guidance to small groups and individual students.
Item example:
The students work together with help from the teacher.

c. Cooperation

The extent to which students help and understand each other.

Item example:

The students help each other to find a solution.

d. Taskorientation

The extent to which pupils stay on the task and work with commitment, without quarrelling or getting distracted (time engaged in learning).

Item example:

There is much talking and noise in the class.

For reasons of convenience I will use abbreviations to refer to these four clusters. I will call the two teacher measures Instruction and Guidance and the two student measures Cooperation and Taskorientation (see also the research model fig. 1). All measures are supposed to have a positive effect on learning results. In the analysis the scale ends of "negative" items are reversed e.g. talking, noise, quarrelling. We used the class-means of the perception scores as a measure of what happened in the classroom. It's necessary to keep in mind that the process measures are perception scores and not scores from an outside observer. Tabel 1 gives an indication of the reliability of the measures.

Percia-cluster	number of items	∝	R	F
a. Whole class-instruction	4	.48	.89	8.2 sig.0000
b. Guidance of small groups	3	.46	-.05	1.4 sig.1377
c. Cooperation	8	.57	.74	4.9 sig.0000
d. Task-orientation	4	.57	.88	9.6 sig..0000

Table 1. *Reliability coefficients PERCIA-clusters*

The first column presents the number of items in the subscales. In the second column are shown the alpha-coefficients, they are low. This is not surprising. The numbers of items in each subscale are small (3-8) and the correlations between the items in each subscale are low (.02 to .40).

For measuring differences between classes, the intra-class correlation coefficient of Horst (R in the third column) is a better indicator. These R's give an indication of the reliability of class means. R gives an indication of the extent to which subscales discriminate between classes in relation to differences within classes. The R's indicate that the means are reliable measures except for guidance.

If we look in the column (F) we find the same pattern. The F is an indicator of the ratio of the differences between groups versus the differences within groups. The F (analyses of variance) also indicates that this "Guidance" cluster is not a good measure to differentiate between classes. From tabel 1 I conclude that the PERCIA-means are reasonably reliable measures except for "Guidance".

Table 2 and 3 give the means and standard deviations per class and per school.

	Teacher				Students				N
Percia	Instructions		Guidance		Cooperation		Task-orientation		
	M	SD	M	SD	M	SD	M	SD	N
total	15.0	2.9	9.5	2.3	28.3	4.0	10.9	2.9	342
class									
1	13.2	3.2	10.6	2.7	26.9	5.2	9.5	2.1	19
2	15.2	3.1	10.0	2.7	28.6	4.3	9.7	2.7	16
3	15.5	2.4	9.9	1.3	27.8	3.1	10.5	1.9	16
4	14.9	2.7	9.5	1.8	28.9	2.6	9.7	2.0	17
5	12.8	2.6	10.1	2.8	26.1	4.7	7.4	2.6	16
6	17.1	2.8	10.5	2.4	29.8	3.6	10.1	2.2	17
7	14.1	1.6	9.4	2.6	27.1	3.7	7.9	1.9	14
8	9.5	2.6	8.9	3.6	24.3	5.2	8.5	2.5	8
9	13.2	3.0	10.0	2.9	28.3	3.1	11.0	3.2	12
10	15.4	2.9	8.9	2.3	27.6	3.1	10.5	3.6	15
11	16.5	2.1	10.0	2.1	30.4	3.6	12.2	2.4	22
12	16.0	2.4	9.5	1.8	29.0	2.9	12.8	2.5	26
13	16.0	1.9	9.5	1.6	29.3	2.5	12.9	1.9	24
14	16.7	2.1	9.0	1.9	28.5	3.8	11.9	3.2	26
15	16.5	1.9	8.9	1.6	31.1	3.3	12.9	2.3	25
16	15.4	1.5	8.6	2.4	30.8	3.7	13.4	1.6	21
17	13.8	3.1	9.0	2.7	26.2	3.0	9.7	2.3	23
18	12.8	3.1	8.8	2.1	25.1	4.6	10.4	2.6	25

Table 2. *PERCIA-clusters per class: means and standard deviations*

	Teacher				Students				
	Instructions		Guidance		Cooperation		Task-orientation		N
	M	SD	M	SD	M	SD	M	SD	
total	15.0	2.9	9.5	2.3	28.3	4.0	10.9	2.9	342
school 1	14.4	3.2	9.9	2.5	27.7	4.1	9.5	2.7	150
school 2	15.5	2.7	9.2	2.1	28.7	4.0	12.0	2.7	192

Table 3. *PERCIA-clusters by school: means and standard deviations*

All differences between schools in table 3 are significant (.00). School 2 has a significant higher mean on all PERCIA-clusters except on guidance. This means that school 2 has (a)more teacher instruction and reflection in the class as a whole, (b) better cooperation, (c) more discipline and taskorientation and (d) less guidance of small groups or individual pupils by the teacher, compared with school 1. These differences between processes in the two schools reflect the striking differences between the two student populations. Students characteristics may have significant influences on the processes in the classrooms.

Table 4 presents the intercorrelations between the four measures on class level. (These measures can be seen in the middle rectangle of the research model in figure 1.)

PERCIA clusters	INSTRUCTION	GUIDANCE	COOPERATION
Instruction			
Guidance	.12 p=.32		
Cooperation	.85 p=.001*	.12 p=.33	
Task-orientation	.63 p=.002*	-.28 p=.13	.73 p=.001*

Table 4. *Intercorrelations between PERCIA-clusters (n=18)*

From table 4 the following conclusions can be drawn:

1) In classes where the teacher frequently gives instruction to the class as whole, the cooperation between students is more frequent as compared to students in classes where the teacher gives less instruction. (Instruction and Cooperation =.85).
2) In classes where the teacher frequently gives instruction, the students show more task-orientation (Instruction and task-orientation=.63)

3) In classes where the students show a strong task-orientation one sees more frequent cooperation between students compared with classes which have less task-orientation, (less time on task, more quarrels). (Task-orientation and cooperation =.73).

Notice that all correlations, where "Guidance" is involved, are low and not significant.

5. OUTCOMES

Before the implementation of the curriculum a pretest was administered to all students. After the implementation a post-test followed. Pretest and post-test are parallel versions of a curriculum-specific test in mathematics. The items of the test consist of mathematical problems in real-life situations. Pretest and post-test have alpha's of .72 and .79. Table 5 gives the mean results.

	Pre-test		Post-test		Difference-score		N
	M	SD	M	SD	M	SD	
Total	25.7	(10,5)	34.6	(12.4)	8,9	(8.2)	349
Schools							
1	19.7	(8.9)	27.7	(9.6)	8.0	(9.1)	155
2	30.5	(9.0)	40.1	(8.3)	9.6	(7.4)	194

Table 5. *Mean scores: pre-test, post-test and gain scores (difference-score)*

As can be seen the mean gain score between pretest and post-test about 9 points. School 1 has a gain score of 8 points, the gain score for school 2 is 9.6 points. There is a significant difference in the mean scores on pretest and post-test between the two schools: school 2 has higher scores.

What about the distribution of the scores? The maximum score on pre- and post-test is 71 points. On the pretest the lowest score was 1 point and the highest was 51 points. On the post-test this interval was 1 to 57 points. Tables 6 and 7 and figures 2 - 7 give more information about the distribution of the scores.

	PRE-TEST			POST-TEST		
score interval	school 1	school 2	total	school 1	school 2	total
1- 4	4	-	4	2	-	2
5- 9	13	3	16	2	-	2
10-14	42	7	49	13	1	14
15-19	32	16	48	22	1	23
20-24	28	24	52	28	8	36
25-29	27	45	72	28	12	40
30-34	15	38	53	34	21	55
35-39	9	35	44	23	44	67
40-44	3	20	23	16	53	69
45-49	-	10	10	3	36	39
50-54	-	2	2	-	15	15
55-59	-	-	-	-	8	8
	n=173	n=200	n=373	n=171	n=199	n=370
	M=19.8	M=30.4	M=25.5	M=27.1	M=40.0	M=33.4
	SD= 9.0	SD= 9.1	SD=10.5	SD= 9.6	SD= 8.3	SD=12.4

Table 6. *Frequencies of scores on pretest and post test per school*

Rance of difference scores	school 1	school 2	totaal
-13 to - 9	4	3	7
- 8 to - 4	13	2	15
- 3 to + 1	19	25	44
2 to 6	34	35	69
7 to 11	32	57	89
12 to 16	26	37	63
17 to 21	19	22	41
22 to 26	5	13	18
27 to 31	1	-	1
32 to 36	2	-	2
	n=155	n=194	n=349
			M=8,87
			SD=8.20

Tabel 7 *Frequencies of difference scores per school*

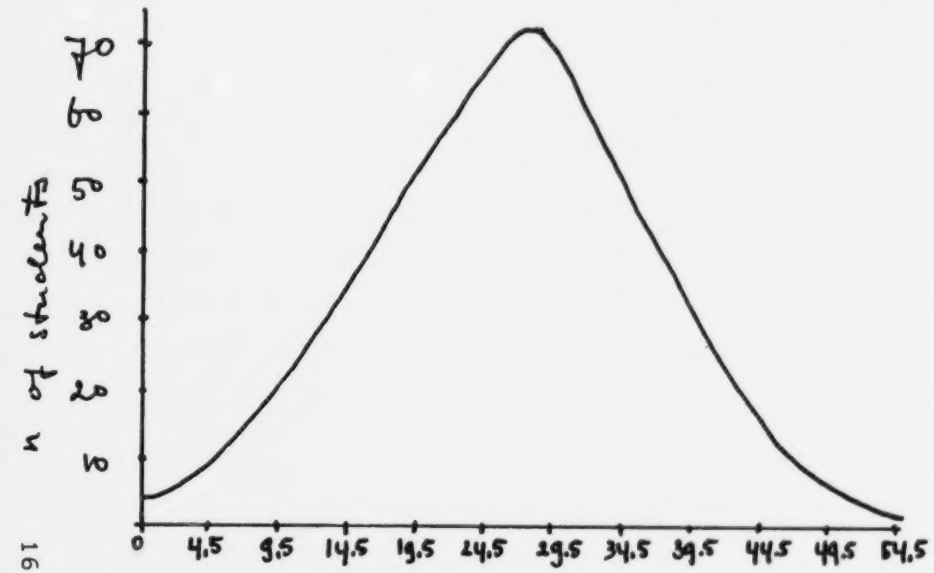

Fig 2 scores pretest total (n=173)

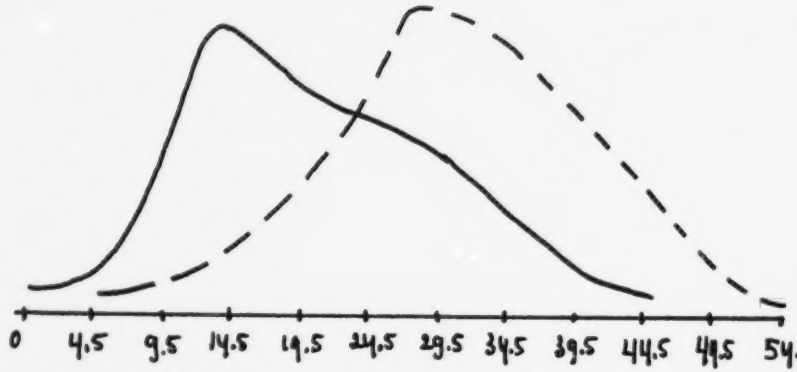

Fig. 3 scores pretest school 1 (n=173) and school 2 (n=200)

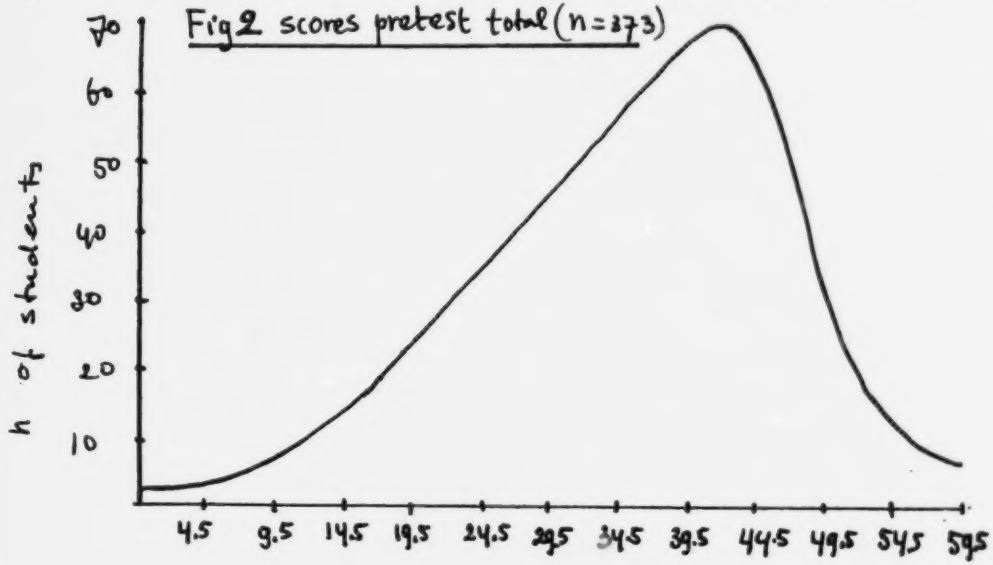

Fig. 4 scores post-test
n = 370 (total)

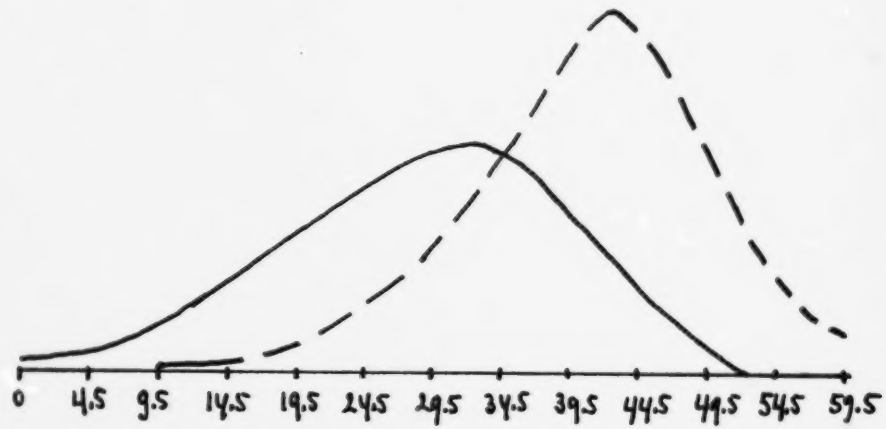

Fig. 5 scores post-test per school
school 1 (n=171) school 2 (n=199)

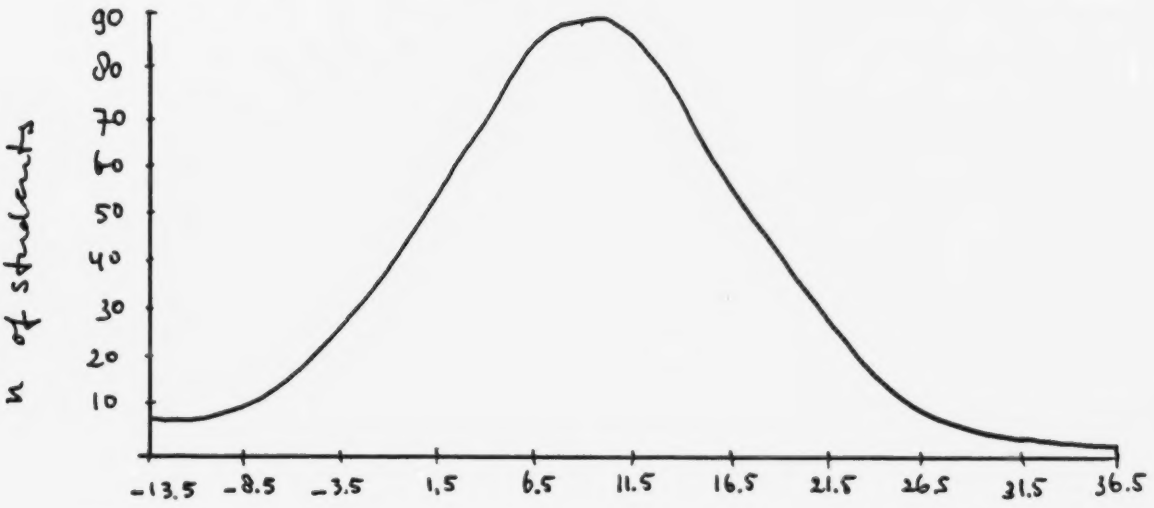

Fig. 6 Difference scores pretest-posttest
(total n=349)

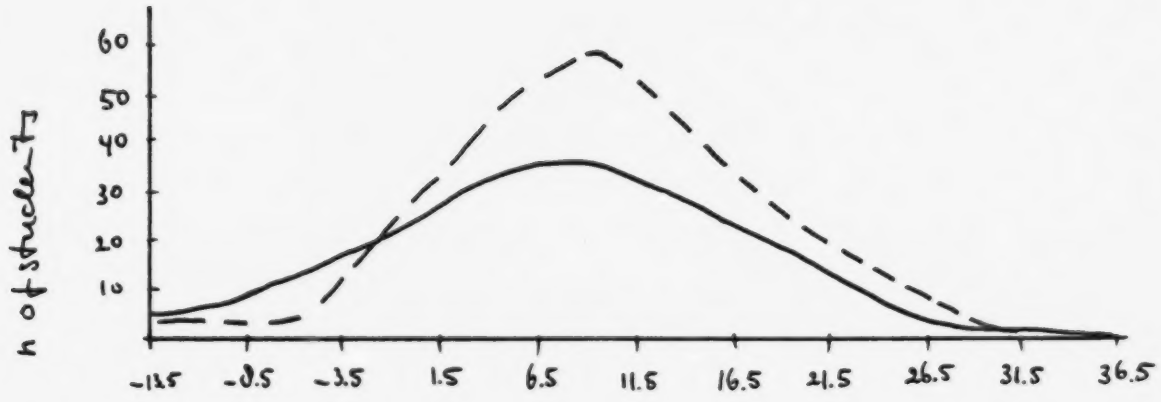

Fig. 7 Difference scores per school

school 1 ———— (n=155)
school 2 - - - - - - - - (n=194)

From the outcomes as measured bij the pre-test and the post-test the following conclusions can be drawn:

1) There are great differences between students and between schools in scores on prestest and post-test.
2) On the average, the students make a significant progess from pretest to post-test (9 points, which is about one S.D.).
3) Not all students reach an acceptable minimum level at the post-test.
4) There are 66 students (almost 20%) who did not make a significant progress.

6. PROCESSES AND OUTCOMES

From our general hypothesis, four specific hypotheses about teacher and student activities in relation to outcomes were derived.

I. Instruction

In classes in which the teacher gives much systematic instruction to the class as a whole, learning results will be better than in classes with less instruction.

II. Guidance

In classes in which the teacher gives much guidance to small groups and individual pupils, learning results will be better than in classes with less guidance.

III. Cooperation

In classes in which students work together and understand each other, learning results will be better than in classes with less cooperation between students.

IV. Taskorientation

In classes in which students work with a good discipline and a relatively high score on 'time on task', learning results will be better than in classes with less task orientation.

For operationalisations see paragraph 4 of this paper.

Table 8 gives the correlations which are mentioned in the four hypotheses above.

	PRETEST	TEACHER		STUDENT		POST-TEST
		Instruction	Guidance	Cooperation	task-orientation	
PRETEST		.37 p=.07	- .52 * p=01	.37 p=.07	.66 * p=.001	.88 * p=.001
POST-TEST	.88* p=.001	.35 p=.08	-.35 p=.08	.38 p=.06	.74* p=.002	

Table 8. *Correlations between processes and outcomes (classlevel n= 18)*

From the figures in tabel 8 the following conclusions may be drawn.

1. Whole class instruction (I), Cooperation (III) and Taskorientation (IV) show a correlation in the expected direction. Only (taskorientation (IV) shows a significant correlation with learning results (post-test). In this sense we may say that our general hypothesis is confirmed in relation to several aspects.
2. Guidance of small groups and individual pupils (II) give a negative correlation with the post-test. This was not expected.

3. The highest correlation in table 8 is the correlation between pretest and posttest: .88 (on class level n=18). On pupil

level we found a correlation between pretest and post-test of .70 (n=342). Thus it follows that prior knowledge is a strong 'predictor' of the learning results at the end. In addition, table 8 shows that in all cases the correlations between the four process measures and the post-test are accompanied by correlations of the same magnitude, between pretest and these processes. Pretest is also a 'predictor' of processes in the class. This fact raises the question whether or not the process measures actually explain any variance in the post-test.

To get an answer on this question we undertook a multiple regression analysis.

In order to be able to carry out a regression analysis on pupil level we assigned each student the mean class score of the process measures (mean perception score).

The assumption behind this procedure is that the mean perception score of a class is a measure of the treatment (independent variable) of the students in the class. For example: every student in class 1 gets a score of 13,2 on Instruction (the mean score of his class).

According to the ordering of variables in our research model we did this regression-analysis on the post-test which may be seen as the dependent variable. (Table 9, 10, and 11).

SUMMARY TABLE

STEP	VARIABLE ENTERED	VARIABLE REMOVED	F TO ENTER OR REMOVE	SIGNIFICANCE	MULTIPLE R	R SQUARE	R SQUARE CHANGE	SIMPLE R	OVERALL	SIGNIFICANCE
1	Instruction		16.89424	.000	.21727	.04720	.04720	.21727	16.89424	.000
2	Guidance		33.60862	.000	.36458	.13291	.08571	-.28233	26.05920	.000
3	Cooperation		5.92313	.015	.38445	.14780	.01489	.24278	19.59873	.000
4	Taskorientation		93.86435	.000	.57708	.33303	.18522	.53630	42.19174	.000

Table 9. Multiple regression analysis with post-test as dependent variable

STEP	VARIABLE ENTERED	VARIABLE REMOVED	F TO ENTER OR REMOVE	SIGNIFICANCE	MULTIPLE R	R SQUARE	R SQUARE CHANGE	SIMPLE R	OVERALL F	SIGNIFICANCE
1	Pretest		330.47507	0	.70154	.49216	.49216	.70154	330.47507	0
2	Instruction		2.38927	.123	.70406	.49571	.00354	.21727	167.10537	.000
3	Guidance		.36208	.548	.70445	.49624	.00054	-.28233	111.31525	.000
4	Cooperation		2.59424	.108	.70716	.50008	.00384	.24278	84.52762	0
5	Taskorientation		59.85495	.000	.75860	.57548	.07540	.53630	91.36792	0

Table 10. Multiple regression analysis with post-test as dependent variable (corrected by pretest)

STEP	VARIABLE ENTERED	VARIABLE REMOVED	F TO ENTER OR REMOVE	SIGNIFICANCE	MULTIPLE R	R SQUARE	R SQUARE CHANGE	SIMPLE R	OVERALL F	SIGNIFICANCE
1	School		169.00141	.000	.57565	.33137	.33137	.57565	169.00141	.000
2	Pretest		169.53097	.000	.74420	.55384	.22246	.70154	211.02861	.000
3	Instruction		.04800	.827	.74425	.55390	.00006	.21727	140.30782	.000
4	Guidance		11.67388	.001	.75419	.56880	.1489	-.28233	111.46267	.000
5	Cooperation		3.27352	.071	.75693	.57294	.00415	.24278	90.42464	0
6	Taskorientation		13.37360	.000	.76765	.58929	.1635	.53630	80.34956	.000

Table 11. Multiple regression analysis with post-test as dependent variable (corrected by school and pretest)

Errata: tabel 11, R-Square change .1489 → .01489
.1635 → .01635

Results regression analyses

Appendix by table 9, step 4, Beta-coefficients

	Beta	Sign. of F
1 Instruction	-.072	.381
2 Guidance	-.016	.761
3 Cooperation	-.236	.013
4 Task-orientation	.748	.000

Appendix by table 10, step 5, Beta-coefficients

	Beta	Sign. of F
1 Pretest	.579	.000
2 Instruction	-.096	.145
3 Guidance	.120	.006
4 Cooperation	-.176	.020
5 Task-orientation	.497	.000

Appendix by table 11, step 6, Beta-coefficients

	Beta	Sign. of F
1 School	.223	.001
2 Pretest	.553	0
3 Instruction	-.141	.034
4 Guidance	.169	.000
5 Cooperation	-.064	.434
6 Task-orientation	.309	.000

If we look at e.g. the R-square changes and Beta-coefficients in tabel 9, 10 and 11 we may conclude that only task-orientation gives a relatively strong, significant contribution -in the expected direction- to the post-test. Thus only hypothesis is IV is confirmed in all regression analyses. The contribution of the other three process measures is small and not always in the expected direction. May be instruction and cooperation give a positive contribution to the post-test via task-orientation, (multicollinearity).

All contributions of process measures in classes remain far behind the contribution of pretest to the variance on the post-test. In addition to this, the factor school (1 or 2) accounts for relatively strong effects. Thus it is shown that school and pretest are the main determinants of variance in the post-test. (together 55% of the variance)

7 SUMMARY AND CONCLUSIONS

1. There are differences in implementation of the curriculum in the several classes; this means that there are differences in teaching activities (Instruction and Guidance) between teachers and differences in student activities (Cooperation and Task Orientation) between classes.
2. There are striking differences in outcomes on the pre-test and the post-test between students, classes and schools. Not all students reach an acceptable minimum-level and about 20% of the students seems to make no progress at all. School 1 has a mean level on the post-test which is lower than the mean level of school 2 at the pre-test!
3. We found high correlations between teacher activities and student activities. In classes where teachers spend much time on activities addressing the class as a whole (instruction, guidance, feedback), the pupils demonstrate (1) substantial cooperative problem solving in groups and (2) less behavior problems and off-task behavior.
4. There is strong correlation between pre-test and post-test on the student level and on the class level. Prior knowledge on Mathematics is a factor of importance for the results on the post-test.

5. The hypothesis about the relation between processes in classes and outcomes is confirmed in relation to only one aspect: task-orientation. In classes with a strong task-orientation (commitment, time on task) the larning results are better than in classes with less task-orientation, (off-task, noise, guarreling).
6. Although the students make significant progress in general, the curriculum does not yet realize the idea of Mathematics for all: not all students reached the common goals. Therefore, adjustments are necessary.
   There are some indications that education in mixed ability classes and mixed ability small groups works better by systematic instruction, management and guidance of the class as a whole by the teacher, and by the ability of students to work

together and understand each other.
The finding that prior knowledge as measured by the pre-test is an important discriminating factor for students to profit from instruction and cooperation, favours an approach that gives more help and learning-time to low performing students.

8 RECOMMENDATIONS

If we look at these conclusions, two recommendations may be made in order to improve the curriculum.
First: A modest improvement can be made if teachers give more attention to instruction, guidance and feedback to the class as a whole, (especially because of the relation between these teacher activities and student activities like cooperation and task-orientation discipline and time on task).
Second: If we look at the big influence of the pretest on the post-test much attention should be payed to the rise of the prior knowledge level in each cycle of the teaching-learning process. It is likely that some students will not have the prerequisite (social and cognitive) skills to learn from instruction and profit from cooperation. More attention should be payed to the dilemma of time and achievement. Some students need more time and help in order to reach the common goals and to prevent that they become 'drop-outs' in the small group. This means that curriculum constructors and teachers should use a combination of learning strategies and principles; cooperative learning, mastery learning, adaptive instruction (Slavin, 1985). Only then we might expect to eliminate divisions between pupils and to diminish variance in outcomes. The constructors of the curriculum did underestimate the dilemma of time and achievement (Arlin, 1984). Therefore they did not provide the teachers with curriculum materials to deal with this dilemma in the classroom. If we accept the fact that low performing students need constantly more learning time we probably approach Mathematics for all. This implies a curriculum with built-in differentiations. For example: the use of peer teaching in remedial phases, (after frequent

individual diagnosis) and the use of the computer to registrate the progress of every individual learner. This means a combination of open, informal education in small groups and more structured ways of instruction and remediation, with a strong emphasis on individual accountability and group reward.

9. DISCUSSION

The conclusion: "not all students reach an acceptable minimum-level" (this means the common goals) is arbitrary.

Although the researchers asked again and again for criteria of success or failure, the curriculum-constructors did not provide them before the implementation of the curriculum. When seeing the research data they were more or less disappointed about the results.

Because the curriculum-constructors did not provide an evaluation-criterium in advance, I tried to formulate a criterium afterwards.

If we look at the frequency distribution, take in account the mathematical content of the post-test, and consider the meaning of mathematics for all, we may say that students with a score on the post-test below 25 do not reach the common goals. This is about 20% of the students. Even when we use a lower criterium (e.q. below 20 points) there remains 11% of the students who do not reach this common goal. These students will probably face seriuos problems in mathematics in the future. Thus one may say the dream "Mathematics for all" is not yet realized in this phase of the experiment.

Because of the exploratory character of our research, we may not assume causality. We did not use control groups, so it is possible that not only a teacher "makes or breaks" a class but that the reverse is perhaps also true! The recommendations lack an experimental basis to prove this.

The reliability of our instruments was reasonable except the reliability of one measure (Guidance).

Our process measure are perceptions of students. This is not the same as observing events by a skilled observer. Actually, whole class instruction means students perception of whole class instruction! Student perceptions can be determined by needs for e.g. instruction. Needs may be related to prerequisite knowledge and skills. Thus the differences, e.g. between school 1 and 2, in mean scores on the PERCIA-scale may be due to differences in needs of the students of the two schools. These meeds may, "distort" their perceptions about what actually is going on in the class.

It is possible that regression effects have played a role in the test-results. These effects also might have influenced the correlations.

Difference-scores are measures with great "reliability" problems. Therefore caution is necessary in all cases were difference-scores are involved.

In the regression analysis I have made an transition from the class level to the individual (student) level by giving each student the mean perception score of his class. This may be a matter for discussion.

In addition to the results from the questionaire and the tests we carried out observations and we undertook an a theoretical and content analysis of the curriculum. (mathematical content, didactics, sequency principles). The results of these observations and analysis proved to be helpful in interpreting the research data. From these observations and analyses we were able to give explanations and specific suggestions for improvement.

By and large, there is no reason for pessimism about this curriculum- in- development. Most of the students make significant progress and reach the goals. They like this math, and they enjoy cooperation in small groups. Notice that among the students who did not reach the minimum level are students of very low start-competencies and unfavorable learning conditions. A curriculum from october until april can not counterbalance years of failure in mathematics. I would suggest to make mathematics for all' a reality by giving these students more time and help.

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